1. Charmonium

1.1. History

Prior to 1974, the existence of a fourth quark was a matter of conjecture based on theoretical debate and circumstantial evidence. Three different quarks were known to exist (up, down, strange) along with four leptons (electron, muon, e-neutrino, μ-neutrino). Based upon symmetry, an additional quark was required in order to complete the quark-lepton doublets shown in Figure 1.1.

$$\begin{pmatrix} e \\ \nu_e \end{pmatrix} \quad \begin{pmatrix} \mu \\ \nu_\mu \end{pmatrix} \quad \begin{matrix} -1 \\ 0 \end{matrix}$$

$$\begin{pmatrix} u \\ d \end{pmatrix} \quad \begin{pmatrix} \\ s \end{pmatrix} \quad \begin{matrix} +2/3 \\ -1/3 \end{matrix}$$

Figure 1.1 : Quark-lepton doublets prior to 1974.

The ratio of electron-positron annihilation rates into hadrons versus that for muons depends to first order on (a) the number of colors each quark has, and (b) the number of quark flavors. Note the increase in Figure 1.2 with the addition of each quark. This ratio was shown to increase past a center of mass energy around 3 GeV, and an additional quark flavor with a +2/3 charge could help account for the difference.[1]

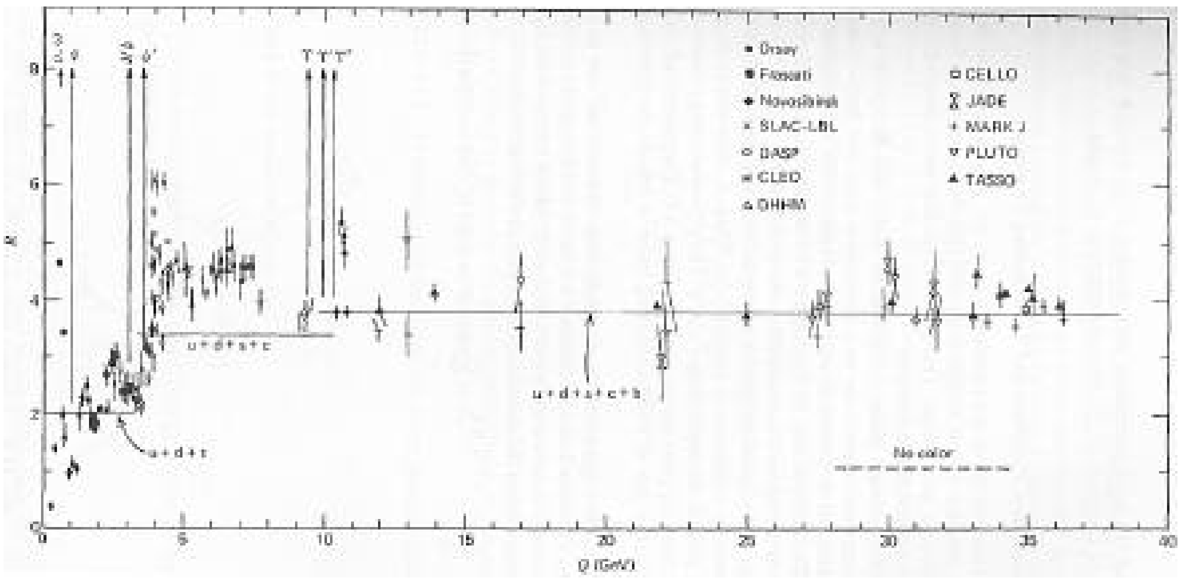

Figure 1.2: Ratio R of e^+e^- annihilation into hadrons from 0 to 40 GeV.[2]

Charm was shown in 1970 under the GIM mechanism to explain the absence of strangeness-changing neutral currents in weak interactions. The suppression of such decays is explained by the different masses of the up and charm quarks.[3] In 1974, Appelquist and Politzer predicted that charmonium would form a set of bound states much like those seen in positronium.[4]

Hence, there were experimental and theoretical indications of the existence of a charmed quark. However, the existence of a fourth quark would mean that there should be more hadronic states. Evidently, if there were a fourth quark, it was heavy enough for these other hadronic states to lie at energies not exceeded by any experiment at that time.

In November, 1974, two experiments independently discovered a resonance with a strikingly narrow total width at around a center-of-mass energy of 3.1 GeV. Brookhaven National Laboratory (BNL) saw this feature in the invariant e^+e^- mass plot produced by collisions of a proton beam on a Beryllium target. They called this new resonance 'J'.[5] The Stanford Linear Accelerator Center (SLAC) saw this resonance in a plot of cross-section vs. energy for multi-hadron final states resulting from e^+e^- annihilations.[6] SLAC called this resonance Ψ , and today it is called the J/ Ψ.

The radial excitation of the J/ Ψ, the Ψ', was soon discovered afterwards.[7] However, these two resonances could not be confidently attributed to the bound state of a charm-anticharm quark pair until the so-called "charmed" mesons (i.e. mesons with a constituent charm quark) were found and the unique behavior of its decays were verified by experiment.[2,8]

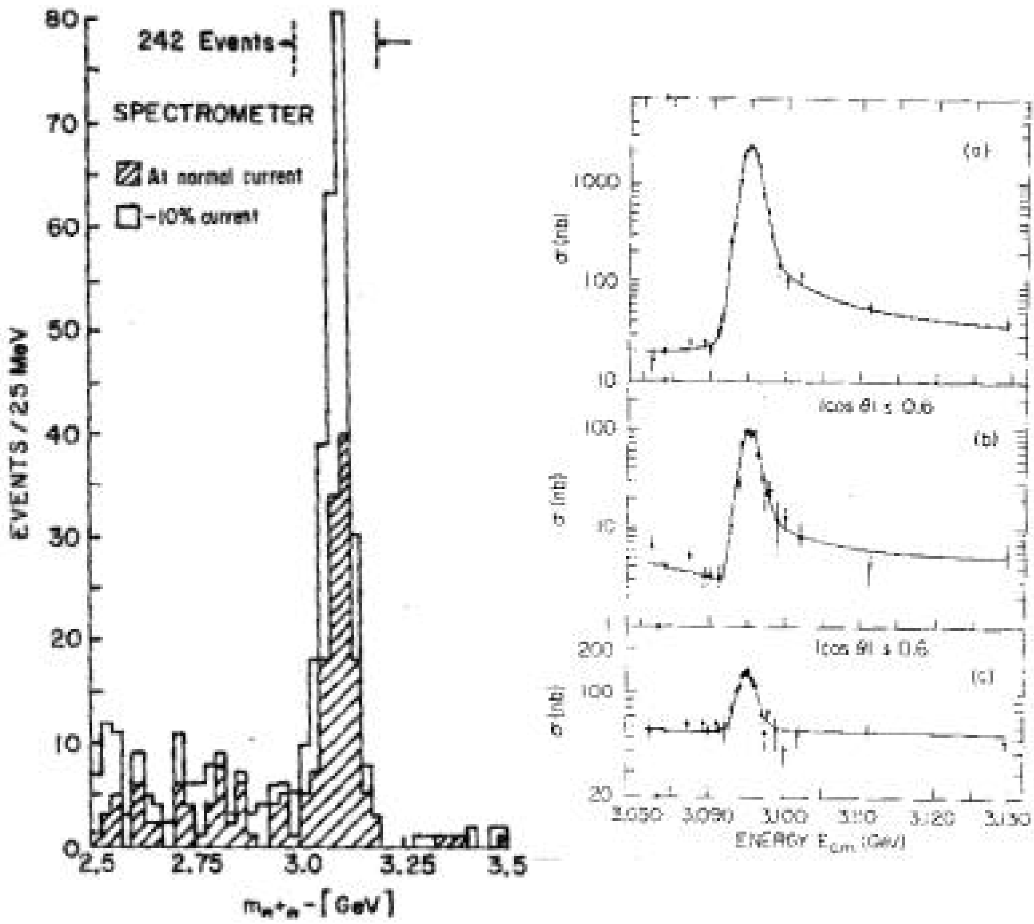

Figure 1.3: Discovery of the J/Ψ at Brookhaven (left)[5], and at SLAC (right).[6]

One may rightfully ask how one can assign these two new resonances to a family called charmonium. The first clue is the extremely narrow widths of these two resonances. This may be explained by the OZI-Rule (Okubo-Zweig-Iizuka), which has been applied to the φ meson. Although the Q-value of the decay (the Q-value is the maximum kinetic energy released in the decay of the resonance to the lowest mass state containing the constituent quarks of the parent resonance) into two K mesons is quite small and the Q-value of the decay into 3 π is much larger, the decay of the φ meson into kaons dominates.[1]

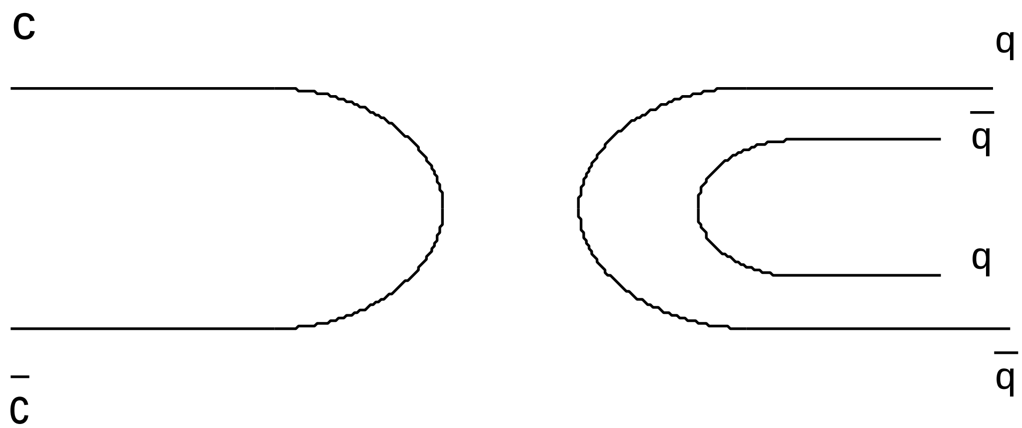

Figure 1.4: OZI-suppressed charmonium decay.

The OZI rule states that the disconnected quark diagrams of Figure 1.4 (where one or more quark lines are not continuous from the initial to the final state) are suppressed relative to the connected quark diagrams of Figure 1.5. In the case of the φ meson, the decay to K mesons is connected and the decay to 3 pions, while open to more phase space, is disconnected (no strange quarks appear in the final state). While the φ meson sits just above the threshold to decay into the two lowest-mass strange mesons, several charmonium resonances are below the threshold for the decay into the two lowest charmed mesons. Decays with connected diagrams are thus not kinematically allowed for charmonium resonances with masses less than twice the mass of the smallest charmed meson, D ($c\bar{u}$). Widths are smaller beneath this threshold because decays are forced to proceed through the suppressed channels.

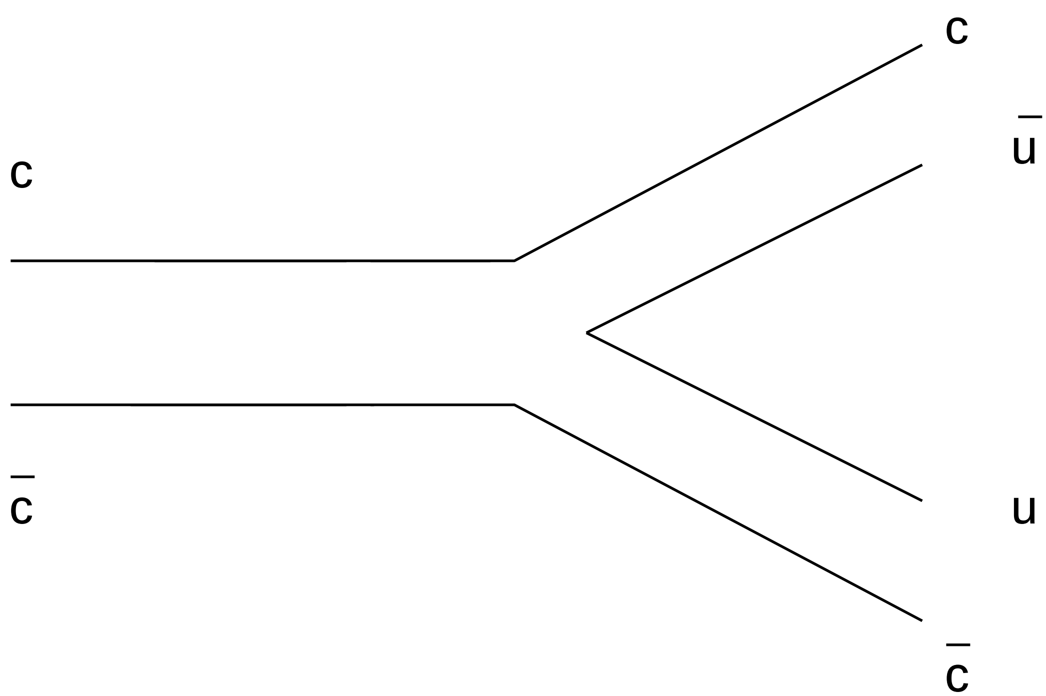

Figure 1.5: OZI-allowed charmonium decay.

The D-meson was discovered at SLAC in 1976 with a mass of 1863 MeV.[9] With the exception of D-wave states that cannot decay into D-mesons due to parity, all charmonium states above twice this mass have broad widths. Since charm is conserved in strong and electromagnetic decays, D-mesons and their charmed cousins must decay via weak interactions, which were subsequently found at SLAC and DESY (Deutsches Elektronen-Synchrotron). Observation of the excited D^* and F mesons, the intermediate P-level states, and the increase in the ratio R mentioned previously near the region of 3.1 GeV all lead one to believe that the Ψ family is built from a charmed quark-antiquark pair.[9]

1.2. Spectroscopy

States in the charmonium spectrum (see Figure 1.6) are labelled either by quantum numbers J^{PC}, where **J** is the total spin of the system ($\mathbf{J} = \mathbf{L} + \mathbf{S}$), or by the notation $n^{2S+1}L_J$, where n is the radial quantum number. For any fermion-antifermion system, the parity quantum number is

$$P = (-1)^{L+1}, \tag{1.1}$$

and the charge-conjugation parity is

$$C = (-1)^{L+S}. \tag{1.2}$$

The quantum numbers for the J/Ψ(3097),[10] Ψ(3686),[11] and the Ψ(3770)[12] have been found to be the same as those of the photon, 1^{--}. Thus these states are readily accessible to production via e^+e^- annihilation, which produces a virtual photon with the same quantum numbers.

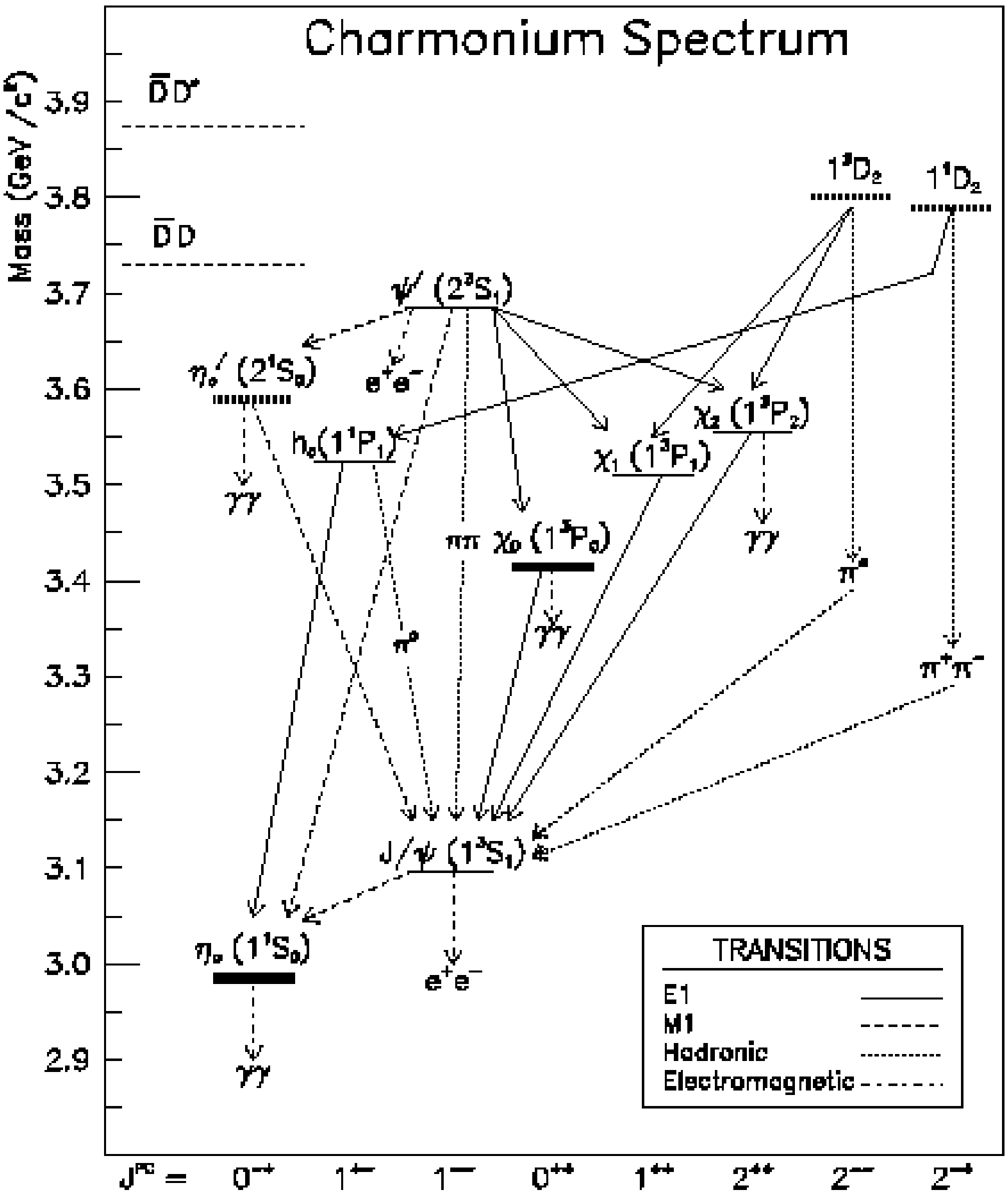

Figure 1.6: The spectrum of charmonium bound states below the threshold to open charm.[13]

The dominating characteristic of the charmonium spectrum is the apparent division between the low-lying states with total widths on the order of 100 keV and higher excitations that have widths on the order of 100 MeV. The principal reason for this effect is the heavy mass of the charmed quark, which is approximately 1.5 GeV. States with center-of-mass energies less than twice the mass of the lightest charmed meson obey the previously mentioned OZI-Rule: The decay into hadrons with lighter quarks is suppressed while the decay into charmed mesons is not allowed.

The other impact of the mass of the charmed quark is that the quantum mechanics of the hydrogen atom can be applied to the charmonium system. One can roughly evaluate the worthiness of the approach by computing the average kinetic energy of charmonium via the virial theorem. Given that a linear term in the potential dominates at the mean radius of the charmonium atom and the expectation value for the kinetic energy,

$$\langle T \rangle = \frac{1}{2} \langle \vec{r} \bullet \vec{\nabla} V(\vec{r}) \rangle , \tag{1.3}$$

where the binding energy of the system is $E_b = 3\langle T \rangle$. Non-relativistically the kinetic energy of two charmed quarks is expressed as

$$\langle T \rangle = 2 \left(\frac{m_c \langle v^2 \rangle}{2} \right). \tag{1.4}$$

As a result, one expects for the charmed quark the square of the velocity to be

$$\langle v^2 \rangle = \frac{E_b}{3m_c}. \tag{1.5}$$

Assuming for the charmonium system that the binding energy is the difference in masses between the J/Ψ (near the lowest possible bound state) and the Ψ' (close to the threshold to open charm), and the mass of the charm quark is approximately 1.5 GeV, one obtains an estimate for this squared velocity of 0.15 c^2. One sees that although relativistic effects may be important, they do not dominate the charmonium system. In stark contrast, the masses of the lighter quarks (from under 10 MeV to a couple hundred MeV) make any non-relativistic treatment unreliable. Spectroscopy of these lower mass states is also encumbered by the sheer number of the light-quark states and their overlapping widths.

1.3. Potential Theory

Since the mass of the charmed quark is so heavy, we attempt to apply a non-relativistic Schrödinger's Equation to the charmonium system with a QCD-inspired potential. Although many different potentials exist, the one most often referred to and the simplest one is the Cornell potential, which addresses the two main concepts in QCD: asymptotic freedom and quark confinement. It consists of a Coulomb-like term representing one-gluon exchange at small distances and a linear term, most likely due to multi-gluon exchanges, that will force quarks to be confined in hadrons and mesons:

$$V(r) = -\frac{4}{3}\frac{\alpha_s(r)}{r} + kr. \tag{1.6}$$

Asymptotic freedom does not exist in QED: The charge of an electron is determined by its long distance behavior. Vacuum fluctuations of electron-positron pairs tend to screen the bare charge, so that the closer you get to the electron, the greater the charge. Whereas photons (the carriers of the electromagnetic interaction) do not carry a charge (and thus do not interact with each other), gluons (the carriers of the strong interaction) carry a *color* charge. As the distance scale becomes larger however, α_s increases as well as the contributions of higher order terms in a perturbative approach.

Hence there is no corresponding long-distance QCD limit for the color charge . In QCD the color charge leads to antiscreening: The closer you get to the particle in question, the less intense the effective force becomes, and the more the particle acts like it were free. The constant α_s and its decrease with increasing energy may be parameterized in terms of its Fourier transform[14]

$$\alpha_s(Q^2) = \frac{12\pi}{(33-2n_f)\ln\left[\frac{Q^2}{\Lambda^2}\right]}, \tag{1.7}$$

where n_f is the number of flavors with mass below Q and Λ is a characteristic scale of about 200 MeV.

Applying such potentials depends in part on the strength of the strong coupling constant α_s. The corresponding coupling constant for QED (Quantum Electrodynamics) is much smaller, 1/137, than the average value of α_s in the charmonium system (typically calculated between 0.2 and 0.3). If α_s is too large, then perturbative techniques, such as describing processes via first-order Feynman graphs, are no longer valid.

The behavior of α_s at large distances is related to the reason why only colorless combinations of subatomic particles occur in nature and quarks are confined to mesons or hadrons. Before a particle with a single color can escape, enough energy exists to manifest new colorless combinations of particles out of the vacuum due to the considerable value of α_s in the QCD potential.

Due in part to the complexities brought about by confinement, a simple non-relativistic potential by itself may not be satisfactory to model the charmonium bound states. Spin dependence of the QCD potential may be accomplished by dividing the above potential into vector and scalar parts, and then using these terms to build a Hamiltonian with spin-spin, tensor, and spin-orbit contributions. The necessity of including spin dependence can be demonstrated by observing the splitting of the S and P state, which would not occur without the relativistic effects inherent in the concept of spin.

In principle, one takes the Bethe-Salpeter equation for a relativistic bound state system and expands everything up to order (v^2/c^2). A non-relativistic reduction of this equation yields the generalized Breit-Fermi Hamiltonian[15]:

$$H = m_1 + m_2 + \frac{\vec{p}^{\,2}}{2\mu} - \frac{1}{8}\left(\frac{1}{m_1^{\,3}} + \frac{1}{m_2^{\,3}}\right) + V(r) + H_{SI} + H_{LS} + H_{SS} + H_T \quad , \tag{1.8}$$

where the potential is a sum of a vector and a scalar contribution,

$$V(r) = V_V(r) + V_S(r) \; . \tag{1.9}$$

The spin-orbit (H_{LS}), spin-spin (H_{SS}), and tensor (H_T) terms may be written as:

$$H_{LS} = \frac{1}{2m^2 r}\left(3\frac{d}{dr}V_V(r) - \frac{d}{dr}V_S(r)\right)\vec{L}\bullet\vec{S} \; ; \tag{1.10}$$

$$H_{SS} = \frac{2}{3m^2}\vec{S}_1\bullet\vec{S}_2\Delta V_V(r) \; ; \text{ and} \tag{1.11}$$

$$H_T = \frac{1}{12m^2}\left(\frac{1}{r}\;\frac{d}{dr}\;V_V(r) - \frac{d^2}{dr^2}\;V_V(r)\right)S_{12} \; , \tag{1.12}$$

where

$$S_{12} = 12\left(\frac{\left(\vec{S}_1 \bullet \vec{r}\right)\left(\vec{S}_2 \bullet \vec{r}\right)}{r^2} - \frac{1}{3}\vec{S}_1 \bullet \vec{S}_2\right). \qquad (1.13)$$

The existence of these spin-dependent potentials has a direct impact on the spacing in the charmonium spectrum. Specifically the hyperfine interaction, H_{SS}, introduces a splitting between the singlet ($^1S_0$) and triplet ($^3S_1$) S-wave resonances(respectively the η_c and the J/Ψ for example). The spin-orbit and tensor interactions also lead to different energy levels for the triplet P-wave states (^3P_J or χ_J , J=0, 1 , 2).

Another concept that is sometimes incorporated into the study of the charmonium spectrum is quantum mechanical mixing, in which every charmonium state is a linear combination of all other charmonium states above and below threshold with the same set of quantum numbers J^{PC} via a “coupled channel “ formalism.[16] The bound charmonium states are not the pure states given by the potential models, but usually the amount of mixing is negligible or small.

In fact, the Ψ(3770) (above threshold) and the Ψ(3686) (below threshold) are probably quantum mechanically mixed.[17] This explains why the decay width of the Ψ(3770) into e^+e^- is too large for a pure D-wave, and the decay width of the Ψ(3686) is slightly too small for a pure S-wave. The amount of S-D wave mixing in each resonance is defined by a mixing angle[18]

$$|\psi(3686)\rangle = |2^3S_1\rangle\cos\theta + |1^3D_1\rangle\sin\theta \qquad (1.14)$$

and

$$|\psi(3770)\rangle = -|2^3S_1\rangle\sin\theta + |1^3D_1\rangle\cos\theta\,, \qquad (1.15)$$

brought about by a tensor interaction[19]

$$\sin\theta \approx 2\sqrt{2}\;\frac{\langle 2^3S_1|\,V_{tensor}|1^3D_1\rangle}{E_{1^3D_1} - E_{2^3S_1}}. \qquad (1.16)$$

Expectations for this mixing angle range from anywhere between 0 to 30 degrees,[20,21] but it is most likely small enough to regard the Ψ' as a pure S-wave resonance.

Of course the true test of all the different potentials in a Schrödinger equation, or any other formalism describing this system, is their ability to replicate and/or predict the behavior of the bound states as found by experiment. This behavior includes such things as the mass of the resonance, its total width, branching fractions, quantum numbers, energy splittings in the system, and the angular distribution of its decay products.

Charmonium serves as an important testing ground for many different aspects of QCD theory and experiment. The angular distributions of $J/\Psi \rightarrow e^+e^-$ and $\Psi' \rightarrow e^+e^-$ are proportional to $1 + \lambda \cos^2(\theta^*)$, where θ^* is the center-of-mass polar angle and λ is the angular distribution parameter. The parameter λ is sensitive to issues such as the wavefunction of the charmonium atom (and therefore the QCD potential), how charmonium couples to $p\bar{p}$, and the structure of the proton at charmonium energies.

Experimentally, the charmonium family has a rich history at several e^+e^- colliders across the globe such as the European Laboratory for Particle Physics (CERN), the Stanford Linear Accelerator (SLAC), the Deutsches Elektronen-Synchrotron (DESY), and the Beijing Electron Positron Collider (BEPC). Recently charmonium experiments have been performed by colliding protons with antiprotons instead of electrons with positrons, first at CERN and then at Fermilab. In the future, heavier bound state systems such as bottomonium and toponium may one day be created by colliding protons and antiprotons based upon the experience gained by examining charmonium.

In summary, the analysis of angular distributions probe different aspects of the QCD potential, and since the $J/\Psi(3097)$ and $\Psi(3686)$ may be produced directly by either e^+e^- annihilation or $p\bar{p}$ annihilation, the study of these resonances in particular serve as an important link in the comparison between the two methods. This thesis is a study of the angular distributions in the exclusive decays of J/Ψ(3097) and Ψ(3686) into e^+e^- via the proton-antiproton annihilation method.