7. Results

In Table 7.1 the final fit parameters to the angular distributions presented in Figures 7.1 and 7.2 are shown. For the process $J/\Psi \to e^+e^-$ in Figure 7.1, the bin size is 0.025 and the range $-0.45 < \cos(\theta) < 0.45$ is fitted. For the process $\Psi' \to e^+e^-$ displayed in Figure 7.2, the bin size of the abscissa is 0.025 and the range $-0.525 < \cos(\theta) < 0.525$ is fitted.

Both angular distributions are fit to a function $A + B\cos^2(\theta)$, yielding the final fit parameters presented in Table 7.1. Significantly, both distributions have a chi-square of 1.0 per degree of freedom.

From this function, the angular distribution parameter is defined as $\lambda = B/A$. With the help of the parameters quoted in Table 7.1, one can derive the statistical error in the angular distribution parameter, δλ. A systematic error is calculated based upon the efficiency of the scintillating fibers (see Figure 6.3), since no correction is made in this regard. For the J/Ψ, the efficiency is always larger than 92% throughout the range being fit. Hence the systematic error for this analysis is estimated as 8% of the angular distribution parameter, 0.05. Likewise for the Ψ', the efficiency is always greater than 96%. The corresponding systematic error is estimated as 4% of the angular distribution parameter, 0.03. The angular distribution parameters and their associated errors for this analysis are summarized in Table 7.2.

Resonance	A	δA	B	δB	χ^2 per d.o.f.
J/Ψ(3097)	282.03	4.2318	177.36	47.828	1.005
Ψ(3686)	59.42	1.7991	38.903	15.040	1.008

Table 7.1: Final Fit Parameters.

Resonance	λ = B/A	δλ (statistical)	δλ (systematic)
J/Ψ(3097)	0.629	0.177	0.05
Ψ(3686)	0.655	0.268	0.03

Table 7.2: Angular Distribution Parameters.

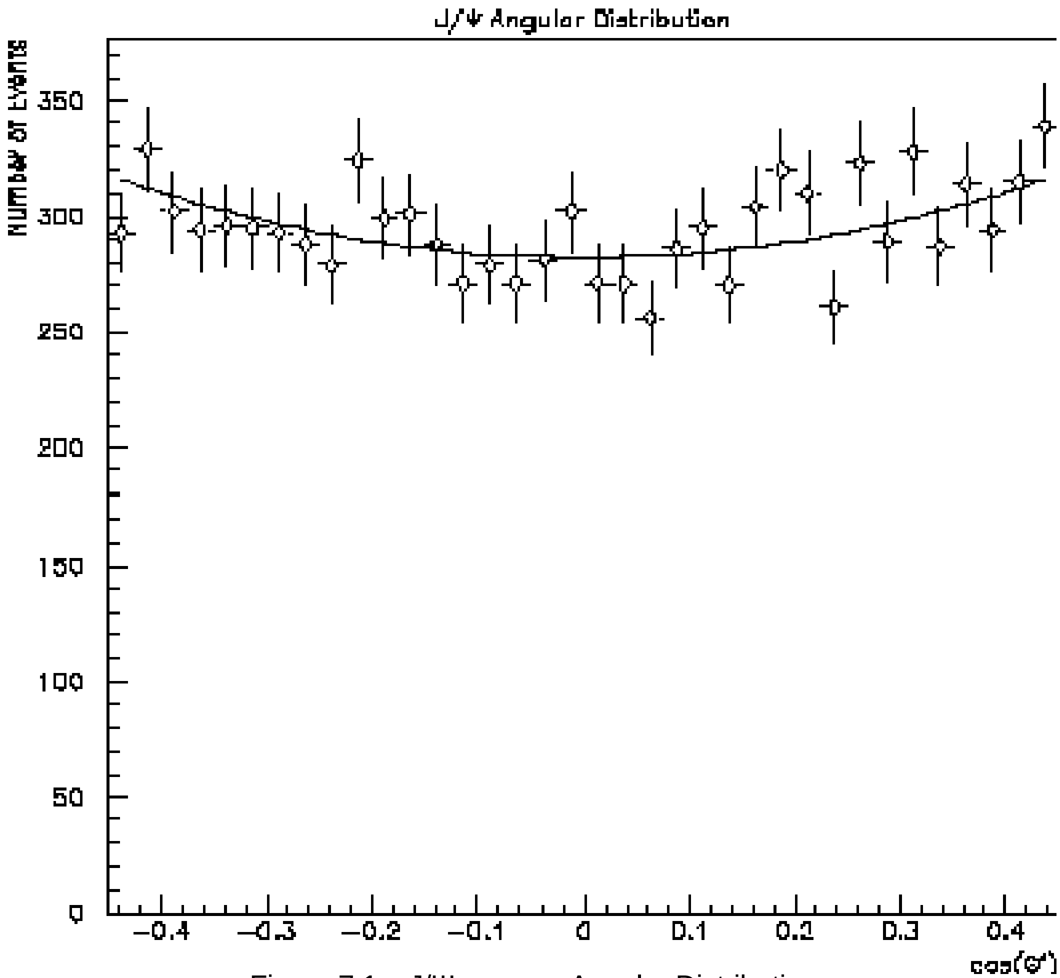

Figure 7.1 : J/Ψ --> e^+e^- Angular Distribution

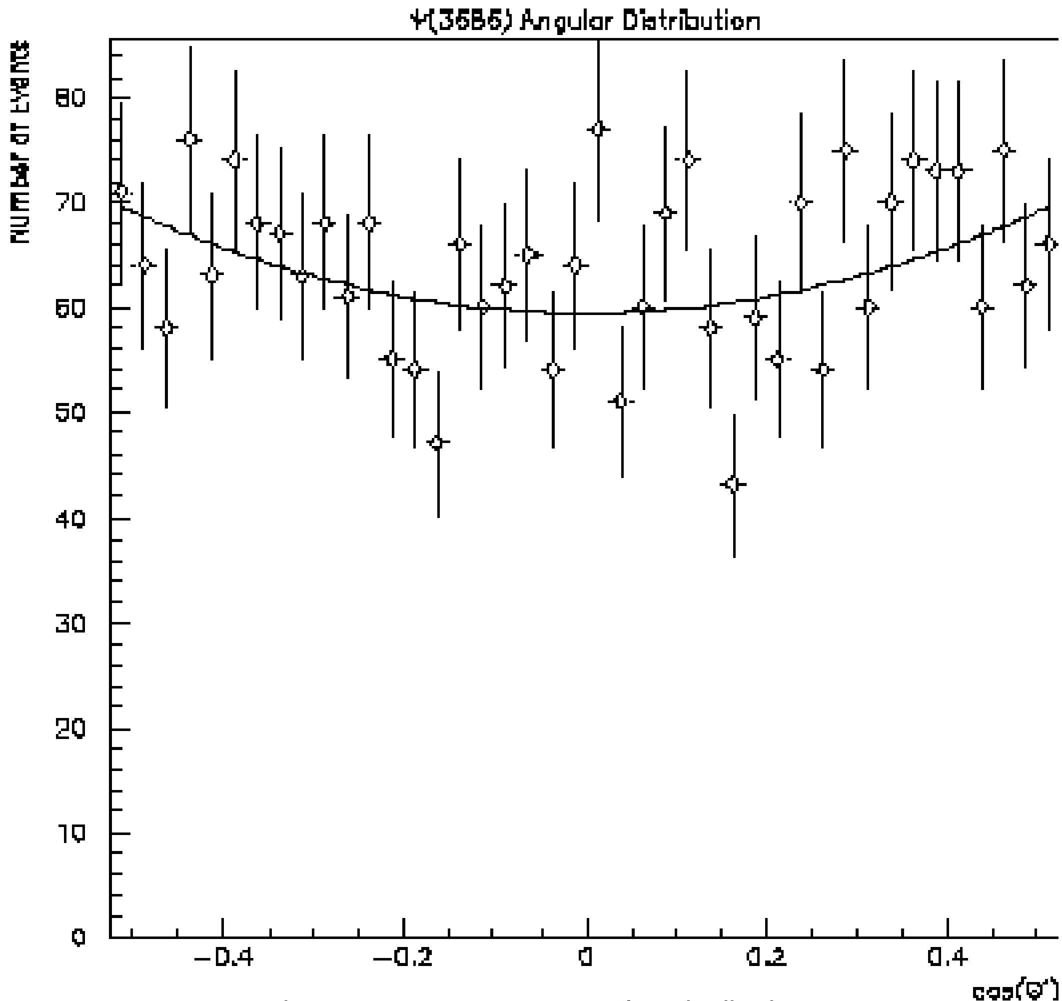

Figure 7.2 : Ψ' --> e^+e^- Angular Distribution

Using these results for the angular distribution parameter, the value for the ratio of the Sachs form factors[76] G_E and G_M dictated by Lorentz invariance at both resonances can be deduced from equation (5.20):

$$\left|\frac{G_E}{G_M}\right|^2 = \left(\frac{E_{CM}{}^2}{4m_p{}^2}\right)\left(\frac{1-\lambda}{1+\lambda}\right). \qquad (7.1)$$

The ratio of these strong form factors is zero when λ is 1. Setting this ratio to one yields λ derived by Claudson, Glashow, and Wise,[82] where the spin-flip amplitude was set to zero.

Furthermore, from equation (5.16), one can determine the probability that the charmonium atom is produced in a helicity one or helicity zero configuration in the helicity formalism:

$$\frac{2\,C_0{}^2}{C_1{}^2} = \frac{1-\lambda}{1+\lambda}. \qquad (7.2)$$

Here $C_1{}^2$ is the probability for the charmonium atom to be produced in a helicity one configuration, and $2C_0^2$ is the probability for helicity zero production. We can then solve for C_1^2 from the normalization condition $C_1^2 + 2\,C_0^2 = 1$.

Resonance	λ	$\|G_E/G_M\|^2$	C_1^2
J/Ψ(3097)	0.63 ± 0.18 ± 0.05	0.619	0.815
Ψ(3686)	0.66 ± 0.27 ± 0.03	0.791	0.830

Table 7.3 : Sachs form factor ratio and helicity one production probability derived from the angular distribution parameter.

As seen in table 7.3, the charmonium 1^{--} resonances are predominately created via the helicity one channel. As the energy scale increases, or more significantly as the ratio of the proton mass to the charmonium mass decreases, helicity one production should increase as well. We note that in e^+e^- annihilation the ratio of the electron mass to the charmonium mass is zero, and the angular distribution parameter for charmonium production by e^+e^- annihilation is 1. Although the angular distribution parameter for the Ψ' is statistically consistent with a value close to one and larger than the same for the J/Ψ, it also appears likely that both resonances share a similar angular distribution parameter. If it is a function of energy, it does not seem to vary much in the charmonium family given the current data.

Historically, only the J/Ψ angular distribution parameter has been measured by experiments. In Table 7.4 a comparison is made with the other experimental values. The thesis value is consistent with the world average for the J/Ψ. Attempts to generate the Ψ' angular distribution parameter have been hindered because the exclusive cross section falls off as Q^{-2}.

Experiment	λ at J/Ψ
Mark I[95]	1.45 ± 0.56
DASP[96]	1.70 ± 1.70
Mark II[97]	0.61 ± 0.23
DM2[98]	0.62 ± 0.11
Mark III[99]	0.58 ± 0.14
Fermilab E760[22]	0.69 ± 0.26
World Average	0.63 ± .08
Fermilab E835(this thesis)	0.63 ± 0.18 ± 0.05

Table 7.4 : Previous experimental results for the angular distribution parameter λ.