Dalitz plot analysis for the $\pi^0\eta\eta$ channel at $\sqrt{S} = 2.98$ GeV/c^2

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1 Introduction

This memo describes the current status of the analysis of the Dalitz plot for the reaction

$$\bar{p}p \rightarrow \pi^0\eta\eta \tag{1}$$

at the center of mass energy of 2.98 GeV. The goal of the analysis is to determine masses and widths of the $\pi^0\eta$ and $\eta\eta$ resonances, and to determine spins of the more prominent resonances.

The event selection and early analysis of the Dalitz plot was performed by Y. Zhang [1].

2 Event Selection

The selection of events corresponding to reaction 1 is described in detail in [2]. A more technical description of all stages of event selection can be found in [3]. The basic idea of event selection was to

a) select events with 6 clusters in CCAL, and no clusters in FCAL;

b) perform kinematic fits to the following four hypotheses

$$\bar{p}p \rightarrow \pi^0\pi^0\pi^0, \tag{2}$$
$$\bar{p}p \rightarrow \pi^0\pi^0\eta\,, \tag{3}$$
$$\bar{p}p \rightarrow \pi^0\eta\,\eta\,, \tag{4}$$
$$\bar{p}p \rightarrow \eta\,\eta\,\eta\,, \tag{5}$$

and select events with the highest fit probability for reaction 4;

Table 1: Numbers of $\pi^0\eta\eta$ events.

$\sqrt{S}$ (GeV)	events
3.0	42980
3.1	8856
3.5	46962
3.6	25977

c) reject events with fit probability less than 10%, reject events if the second best hypothesis has probability greater than 0.01%, and reject events if any photon from an η can be paired with another photon to form a π^0.

Data were taken at $\sqrt{S} \approx 3.0$, 3.1, 3.5 and 3.6 GeV, corresponding to the masses of charmonium states η_c, J/ψ, ${}^1P_1$ and η_c', respectively. The numbers of events after cuts for the four center of mass energies are shown in table 2.

In this memo, only analysis of the data corresponding to the lowest energy point is described. Analysis of the higher energy data, which will be performed later, will provide valuable cross-check of the results at the lower energy.

Figures 1 and 2 show Dalitz plots at 3.0 and 3.5 GeV, respectively, and figures 3 and 4 show $\pi^0\eta$ and $\eta\eta$ invariant mass projections. (Figures 1–4 show raw data, uncorrected for acceptance or efficiency.)

3 Dalitz Plot Density Function

3.1 Kinematic variables

Assuming that reaction 1 proceeds through intermediate resonances (denoted by X), decaying to $\pi^0\eta$ or $\eta\eta$,

$$\bar{p}p \to \pi^0 X \to \pi^0\eta\eta, \quad \text{or} \tag{6}$$

$$\bar{p}p \to \eta X \to \eta\,\pi^0\eta \tag{7}$$

it can be described using four kinematic variables. One possible choice of variables is

$$\theta^*_{\pi\eta_1}, m_{\pi\eta_1}, \theta'_{\pi\eta_1}, \phi'_{\pi\eta_1}, \tag{8}$$

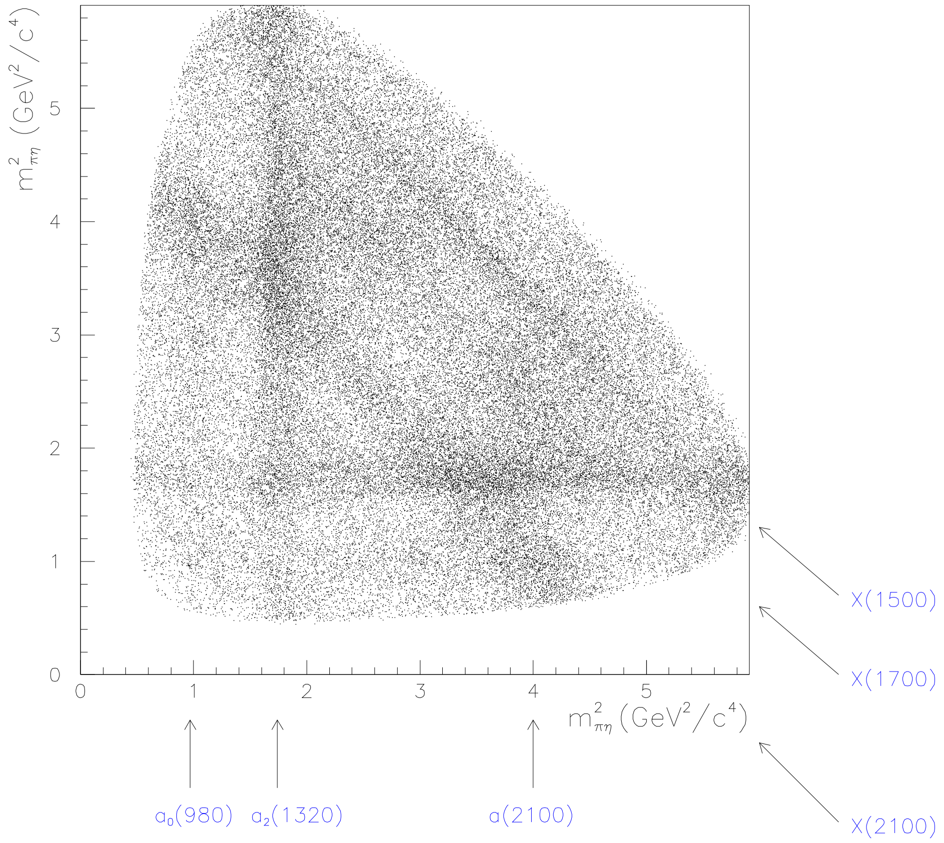

Figure 1: $\pi^0\eta\eta$ Dalitz plot for $\sqrt{S} = 3.0$ GeV.

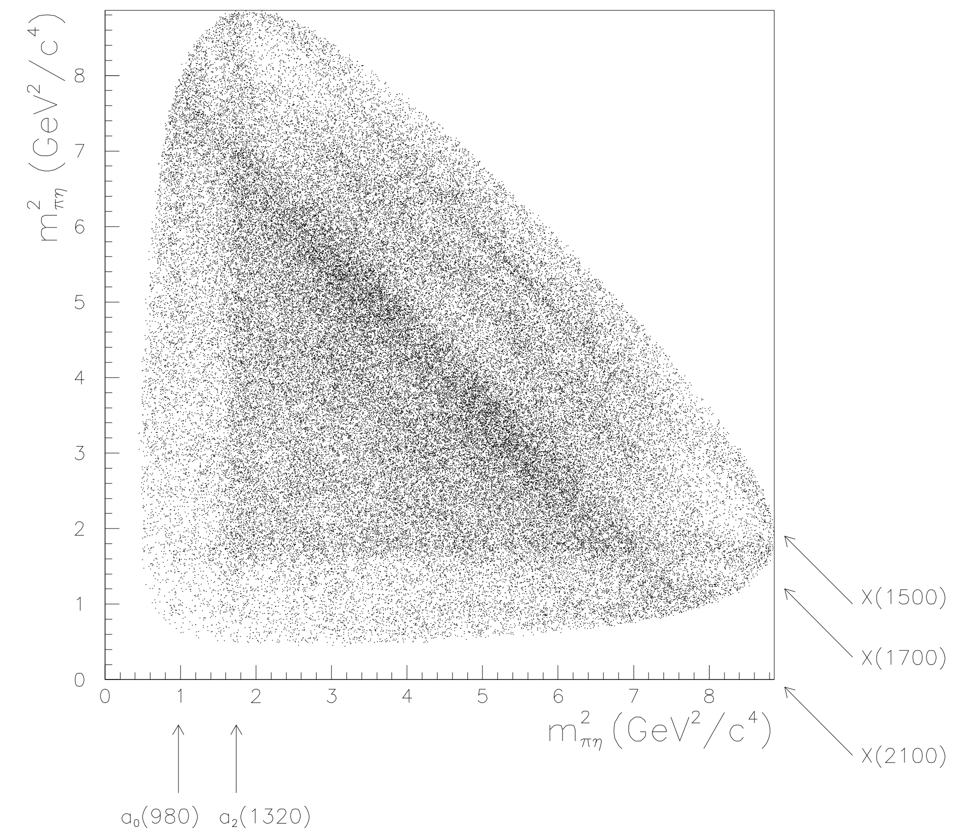

Figure 2: $\pi^0\eta\eta$ Dalitz plot for $\sqrt{S} = 3.5$ GeV.

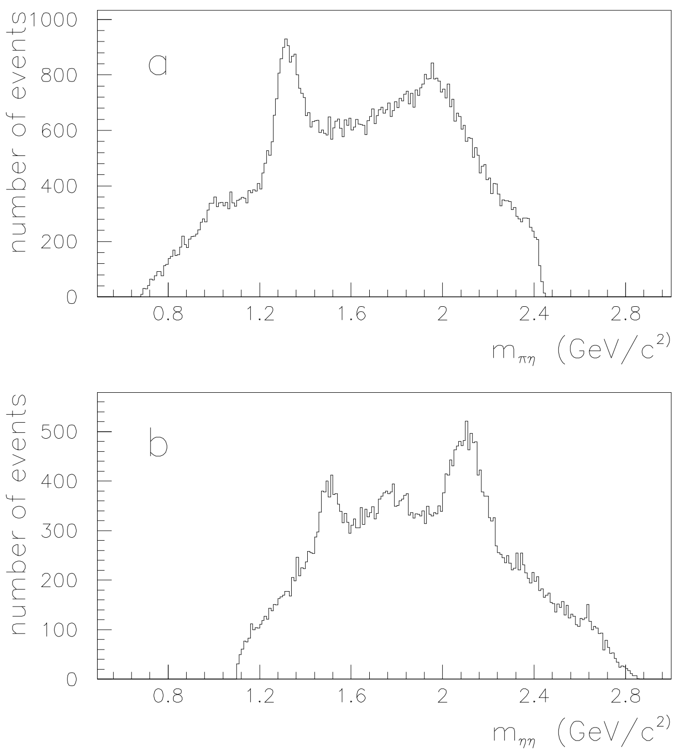

Figure 3: $\pi^0\eta\eta$ invariant mass plot for $\sqrt{S} = 3.0$ GeV.

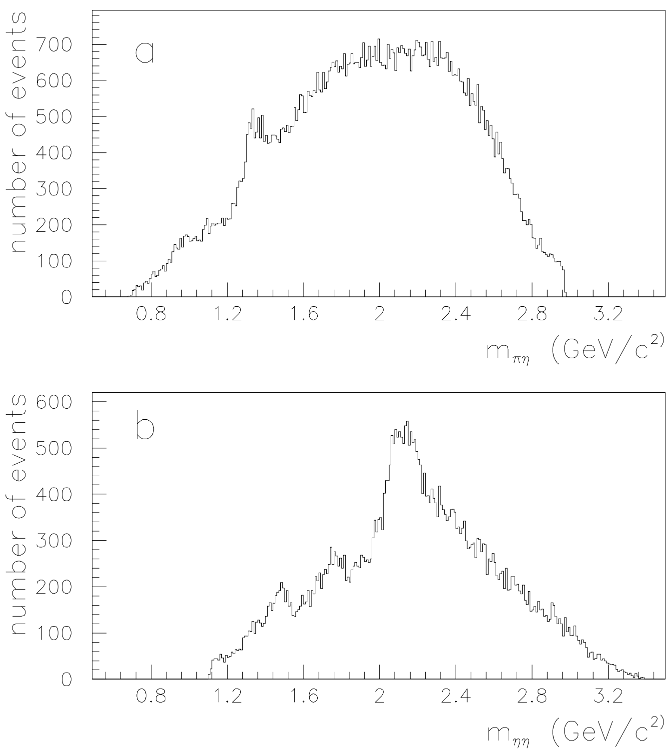

Figure 4: $\pi^0\eta\eta$ invariant mass plot for $\sqrt{S} = 3.5$ GeV.

where $\theta^*_{\pi\eta_1}$ is the polar angle of the resonance X (decaying to $\pi\eta_1$), with respect to the beam direction in the $\bar{p}p$ rest frame, $m_{\pi\eta_1}$ is the invariant mass of the resonance, and $\theta'_{\pi\eta_1}$ and $\phi'_{\pi\eta_1}$ are the polar and azimuthal angles of the η_1 in the helicity frame. (Helicity frame refers to the system where X is at rest, and the Z axis is directed opposite to the direction of the spectator η_2.) From $m_{\pi\eta_1}$ and $\theta'_{\pi\eta_1}$, one can determine $m_{\pi\eta_2}$, and one can choose an equivalent set of independent kinamatic variables

$$\theta^*_{\pi\eta_1}, m_{\pi\eta_1}, m_{\pi\eta_2}, \phi'_{\pi\eta_1}. \tag{9}$$

A Dalitz plot shows distribution of events as a function of $m_{\pi\eta_1}$ and $m_{\pi\eta_2}$, integrated over θ^* and ϕ'.

3.2 Resonance amplitudes

The amplitude for the i-th resonance $X \to \pi\eta_1$, which contributes to the Dalitz plot density function, is written as [5, 4]

$$A_i = \sqrt{\frac{m}{q}} \frac{\sqrt{\Gamma(m)}}{(m_0^2 - m^2) - im_0\Gamma(m)} \sum_{k=0}^{J} c_k P_J^k(cos\theta'), \tag{10}$$

where

m_0, Γ_0, J are mass, width and spin of the resonance,

m is measured invariant mass of the $\pi\eta_1$ pair,

q is momentum of the η_1 in the $\pi\eta_1$ rest frame,

θ' is the polar angle of the η_1 in the $\pi\eta_1$ rest (helicity) frame,

$c_k(k = 1, ..J)$ are complex coefficients, and

$$\Gamma(m) = \left(\frac{q}{q_0}\right)^{2J+1} \left(\frac{m_0}{m}\right) \Gamma_0. \tag{11}$$

The amplitude defined in equation 10 is independent of θ^* and ϕ': the coefficients c_k represent values effectively integrated over these variables.

Amplitudes for resonances decaying to $\pi\eta_2$ and to $\eta\eta$ are constructed analogously. For resonances decaying to $\eta\eta$ (identical particles), only the even values of k are allowed.

3.3 Density Function

The Dalitz plot density function is constructed of resonance amplitudes and of a background term. In the idealized case of full acceptance and perfect resolution, the density function should be the square of the sum of the amplitudes.

$$D_{full\,int} = |\sum_{i=1}^{N_{\pi\eta}} A_i(\pi\eta_1) + \sum_{i=1}^{N_{\pi\eta}} A_i(\pi\eta_2) + \sum_{i=1}^{N_{\eta\eta}} A_i(\eta_1\eta_2)|^2 + D_{bck} \qquad (12)$$

where the three sums correspond to the three pairs of final state particles ($\pi\eta_1$, $\pi\eta_2$, and $\eta_1\eta_2$), and $N_{\pi\eta}$ and $N_{\eta\eta}$ are the numbers of resonances decaying to $\pi\eta$ and $\eta\eta$.

The term D_{bck} represents incoherent background, coming from misidentified events. It is parametrized as follows

$$D_{bck} = p_1 \cdot (m_{\eta\eta} - m_{\eta\eta}^{min})^{p_2} \cdot \exp\left(-p_3(m_{\eta\eta} - m_{\eta\eta}^{min})\right) \cdot (m_{\eta\eta}^{max} - m_{\eta\eta})^{p_4} \cdot \exp\left(-p_5(m_{\eta\eta}^{max} - m_{\eta\eta})\right). \qquad (13)$$

Here, $p_1, p_2, ..$ are real parameters, and $m_{\eta\eta}^{min}$, $m_{\eta\eta}^{max}$ are the minimum and maximum values allowed for the $\eta\eta$ invariant mass.

The density function specified by equation 12 contains all the interference terms, including interference between resonances decaying to the same final state particles (same-channel interference), and between resonances decaying to different final state particles (cross interference). If we wanted to include only the same-channel interference terms, the density function should be written as follows

$$D_{same-ch\,int} = |\sum_{i=1}^{N_{\pi\eta}} A_i(\pi\eta_1)|^2 + |\sum_{i=1}^{N_{\pi\eta}} A_i(\pi\eta_2)|^2 + |\sum_{i=1}^{N_{\eta\eta}} A_i(\eta_1\eta_2)|^2 + D_{bck}. \qquad (14)$$

The density function without any interference terms is written as

$$D_{no\,int} = \sum_{i=1}^{N_{\pi\eta}} |A_i(\pi\eta_1)|^2 + \sum_{i=1}^{N_{\pi\eta}} |A_i(\pi\eta_2)|^2 + \sum_{i=1}^{N_{\eta\eta}} |A_i(\eta_1\eta_2)|^2 + D_{bck}. \qquad (15)$$

These density functions are discussed in Section 6.1.

Table 2: Resonances included in the fit.

resonance	decay	spin	m (fit)	m (PDG)	Γ (fit)	Γ (PDG)
a(980)	$\pi\eta$	0	1008 ± 3	983 ± 2	96 ± 10	57 ± 11
a(1320)	$\pi\eta$	2	1331 ± 1	1318 ± 1	125 ± 4	110 ± 5
a(2000)	$\pi\eta$	4	2138 ± 6	2037 ± 26	$1000\pm$	427 ± 120
X(1500)	$\eta\eta$	2	1510 ± 3	1520 ± 7	118 ± 11	118 ± 10
X(1700)	$\eta\eta$	0	1709 fixed	1709 ± 5	146 fixed	146 ± 12
X(2150)	$\eta\eta$	4	2143 ± 6	2175 ?	222 ± 19	150 ± 35 ?
X(2400)	$\eta\eta$	2	2369 ± 27		182 ± 38	
X(2600)	$\eta\eta$	2	$2650\pm$		175 ± 25	

3.4 Resonances included in the fit

It can be shown that if we assume that the resonances, decaying to $\pi^0\eta$ or to $\eta\eta$, are quark-antiquark mesons, then the only allowed J^{PC} combinations are 0^{++}, 2^{++}, 4^{++} etc.

Table 3.4 lists the resonances included in the fit.

4 Acceptance and Efficiency Corrections

Short description of the method, plots.

5 Fitting Method

The maximum likelihood method is used to find the optimal values of resonance parameters. The likelihood function is written as

$$L = \prod_{i=1}^{N_{ev}} \frac{D(m_{\pi\eta_1}, m_{\pi\eta_2}) \cdot \alpha(m_{\pi\eta_1}, m_{\pi\eta_2})}{\int D(m_{\pi\eta_1}, m_{\pi\eta_2}) \cdot \alpha(m_{\pi\eta_1}, m_{\pi\eta_2}) \cdot d\, m_{\pi\eta_1} \cdot d\, m_{\pi\eta_2}}. \tag{16}$$

The product in the equation above runs over all events, and the integral in the denominator is performed over the whole Dalitz plot. The function α is a product of geometric acceptance and detector efficiency.

The likelihood function depends on the resonance parameters (masses, widths, and polarization coefficients) through its dependence on the density function D. In order to find the optimal value of the parameters, the negative logarithm of the likelihood function is minimized.

Once the fit is performed, its goodness is estimated using the Pearson's χ^2 test. The data on the Dalitz plot are divided into 50×50 bins (... non-empty bins), and the χ^2 is calculated as

$$\chi^2 = \sum_{i=1}^{N_{bins}} \frac{(n_i^{data} - n_i^{fit})^2}{n_i^{fit}} . \tag{17}$$

Here n_i^{data} is the number of events in the i-th bin, corrected for acceptance and efficiency, and n_i^{fit} is the number of events in the i-th bin corresponding to the fitted density function.

6 Results

6.1 Study of Interference Effects

a) Comparison of fits using different interference models: (old results)
full interference: $\chi^2/ndf=2.65$
same-channel interf: $\chi^2/ndf=2.08$
no interference: $\chi^2/ndf=2.74$
analogous results were obtained for the $3\pi^0$ *channel*

b) Fit with full interference yields unphysically large widths for most resonances; the other two models yield reasonable widths

c) Study of the region of cross interference of $a_2(1320)$ *with itself: plot mass(*$\pi\eta_1$*) in slices of mass(*$\pi\eta_2$*). A small hint of a shifting mass peak (indicating interference), but numbers of events in the peak are more consistent with no interference than with interference model (need a more quantitative statement).*

d) possible interpretations:
- integration over ϕ' *washes out cross-interference terms*
- detector resolution
- non-physical explanation: more parameters in same-channel interference model hence a better fit
- MC studies could help decide if the first two cases are true

resonance	decay	spin	m (fit)	m (PDG)	Γ (fit)	Γ (PDG)
a_0(980)	$\pi\eta$	0	1009 ± 6	983 ± 2	96 ± 14	57 ± 11
a_2(1320)	$\pi\eta$	2	1334 ± 2	1318 ± 1	137 ± 4	110 ± 5
a (2100)	$\pi\eta$	4	2122 ± 4		≥ 900	
f_2(1270)	$\eta\eta$	2	1275 fixed	1275 ± 5	185 fixed	185 ± 20
X(1500)	$\eta\eta$	2	1508 ± 2	1527 ± 7	96 ± 5	118 ± 10
X(1700)	$\eta\eta$	0	1709 fixed	1709 ± 5	146 fixed	146 ± 12
X(2100)	$\eta\eta$	4	2170 ± 4		282 ± 16	
X(2400)	$\eta\eta$	2	2314 ± 12		237 ± 25	
X(2600)	$\eta\eta$	2	2700 fixed		191 ± 14	

Table 3: Results from the best fit.

e) current conclusion: for now use same-channel interference model. If it is found that the resolution washes out interference effects, go back to no-interference model, or scale down interference terms.

6.2 Best Fit Results

The results of the best fit are shown in Table 6.2 and in Figures 5,6. This fit was performed assuming same-channel interference model. Table 6.2 lists the resonances included in the fit, assumed spin values, masses and widths resulting from the fit, and previously measured [6] values of masses and widths for well known resonances. Figures 5,6 show invariant $\eta\eta$ and $\pi\eta$ mass distributions from the data (corrected for acceptance and efficiency), compared with the distributions corresponding to the fitted density function. Contributions from individual resonances to the fitted function are also shown. The χ^2/ndf for this fit is 1.40 *(1.95 with 30×30 binning.)*.

6.3 Spin Tests

need to redo tests for X(1500) and a2(1320); old tests (with old density fction and acceptance corrections) favor spin 2 or 4 for X(1500) and for a2(1320). Also need cos(θ') plots.

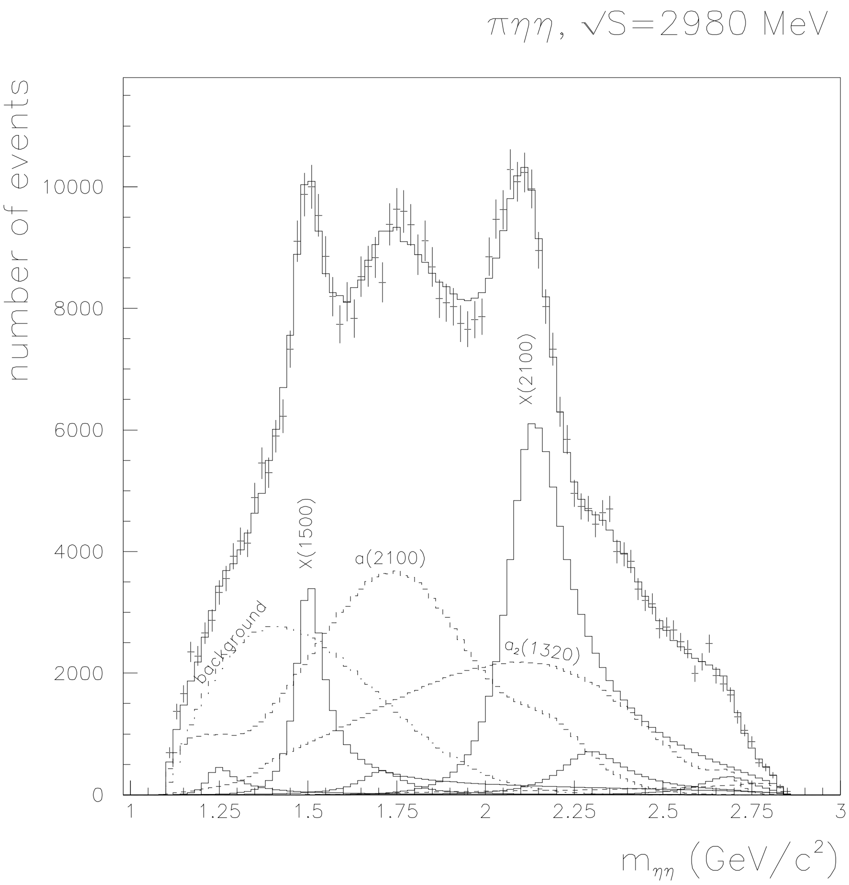

Figure 5: Invariant $\eta\eta$ mass from the data, corrected for acceptance and efficiency (points with error bars), compared with the distribution corresponding to the best fit (histogram); contributions from individual resonances to the fitted function are also shown.

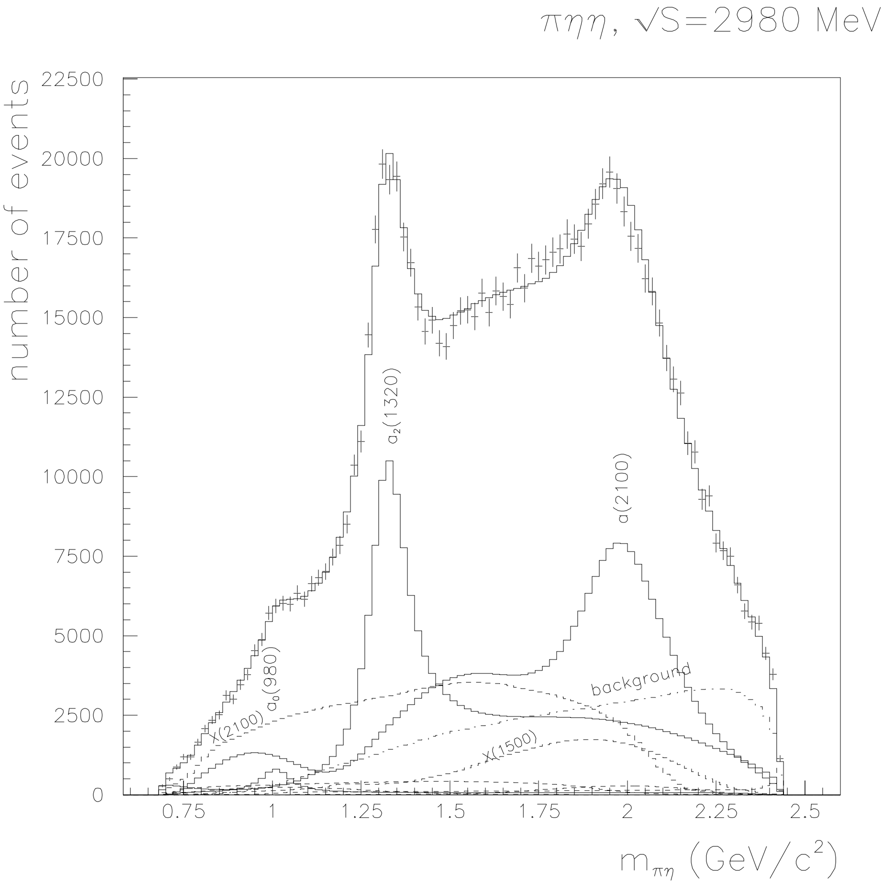

Figure 6: Invariant $\eta\pi$ mass from the data, corrected for acceptance and efficiency (points with error bars), compared with the distribution corresponding to the best fit (histogram); contributions from individual resonances to the fitted function are also shown.

7 Discussion and Conclusions

7.1 Comparison with the Crystal Barrel results

7.2 Conclusions

A Data files, source code, etc.

A.1 Data files

The files containing Dalitz plots and mass projections histograms can be found in `fn760a: amm/lqs/pi2eta/etac_pi2eta.paw`,
`jpsi_pi2eta.paw`,
`1p1_pi2eta.paw` *and*
`etacp_pi2eta.paw`.

A.2 Source code

A.3 This memo

This memo can be found in the following latex file
`fn835z: amm/lqs/text/pietaeta_memo.tex`.
The figures are encapsulated postscript files ...,
and they were made using the following PAW macros
... .

References

[1] *Y. Zhang, A Dalitz Plot Analysis of* $\bar{p}p \to \pi^0 + 2\eta$*, E-760 Memo #298, Dec 1992.*

[2] *T. A. Armstrong et al., "Production of* $\eta\eta$ *Resonances in Antiproton-Proton Annihilations at* $2950 < \sqrt{s} < 3620$ *MeV/*c^2*", Phys. Lett. B 307, 394 (1993).*

[3] *Y. Zhang, About Neutral Event Analysis, PSU note, Feb 1993; also in* `fn760a: yangling/neutr/AAAREADME.TXT`.

[4] *E.Aker et al., Phys. Lett. B 260, 249 (1991).*

[5] *G. Breit and E. Wigner, Phys. Rev. 49, 519 (1936);*
D. Jackson, Nuovo cimento 34, 1644 (1964).

[6] *Review of Particle Properties, Phys. Rev. D 45, (1992).*