Chapter 9

Determination of α_S

The essence of QCD is embodied in its coupling constant, α_s. Measurements of α_s at low momentum transfers are extremely important for the understanding of strong interaction physics. At the Lepton-Photon '93 Conference, Frank Wilczek commented that "if you want to be quantitative about α_s, then the low energy determinations have a big advantage." [100] In other words, Wilczek was making the point that high q measurements of α_s (those in the vicinity of $q = M(Z^0) \simeq 91.2$ GeV) are simply not sensitive to how the strong coupling constant runs, as are measurements of α_s in the low q region, near the charmonium and bottomonium resonances.

The strong coupling constant varies rather rapidly with q at low energies, and 'slows down' at higher energies. Its variation with q is given by [2]

$$\alpha_s(q) = \frac{4\pi}{\beta_0 ln(\kappa)} \times \left(1 - \frac{2\beta_1}{\beta_0^2}\frac{ln(ln(\kappa))}{ln(\kappa)} + \frac{4\beta_1^2}{\beta_0^4 ln^2(\kappa)} \times \left[(ln(ln(\kappa)) - 0.5)^2 + \frac{\beta_2\beta_0}{8\beta_1^2} - 1.25\right]\right) \tag{9.1}$$

where $\beta_0 = 11 - (2/3)n_f$, $\beta_1 = 51 - (19/3)n_f$, and $\beta_2 = 2857 - (5033/9)n_f + (325/57)n_f^2$, in which n_f is the number of quark flavors with masses less than q. κ is shorthand for $q^2/(\Lambda_{\overline{MS}}^{n_f})^2$, and $\Lambda_{\overline{MS}}^{n_f}$ is the QCD cut-off parameter defined in the $\overline{MS}$ renormalization scheme, for n_f flavors. The PDG98 gives the current world averages of $\Lambda_{\overline{MS}}^{(4)} = 305 \pm 25 \pm 50$ MeV, and $\Lambda_{\overline{MS}}^{(5)} = 219^{+25}_{-23}$ MeV. [2] A value of α_s at one scale μ_1 may be compared to that at another scale μ_2 by taking the ratio of Eq. 9.1 evaluated at μ_1 to that evaluated at μ_2, using appropriate values of n_f and $\Lambda_{\overline{MS}}$ if a flavor threshold or thresholds lie between μ_1 and μ_2. We use Eq. 9.1 in this way to evolve our measured value of $\alpha_s(m_c)$ up to the mass of Z^0.

Both η_c and χ_2 annihilate into two gluons. We are therefore able to derive a value of the strong coupling constant α_s at the charm quark mass m_c, by using both our measurements of the hadronic and two photon partial widths of η_c and χ_2. In order to make this determination, we use the pQCD formulae presented in Sec. 2.5.

9.1 α_s determined by η_c data

As we mentioned in Sec. 2.5, α_s may be calculated in the framework of pQCD by taking ratios of the pQCD predictions for gluonic and electromagnetic annihilation widths of charmonium states. This leads to the cancellation of the unknown charm quark mass and the radial wave function at the origin, and its derivative at the origin.

There are two ways we can use our results for η_c to determine $\alpha_{\rm S}$.

1) If we make the assumption that the wave function at the origin for the 1S charmonium states, J/ψ and η_c, is the same, we can calculate $\alpha_{\rm S}$ by the following:

$$\frac{\Gamma(J/\psi \to e^+e^-)}{\Gamma(\eta_c \to gg)} = \frac{8{\alpha_{\rm EM}}^2}{3{\alpha_{\rm S}}^2} \times \frac{(1-5.3\alpha_{\rm S}/\pi)}{(1+4.8\alpha_{\rm S}/\pi)}. \tag{9.2}$$

As we argued in Sec. 8.6, the gg partial width of η_c is essentially equal to its total width, so we evaluate the above equation by inserting the world average of $\Gamma(J/\psi \to e^+e^-) = 5.26 \pm 0.37$ keV, [2] and our measurement of $\Gamma(\eta_c \to gg) = \Gamma(\eta_c) = 26.9^{+10.8}_{-9.5}$ MeV. Solving Eq. 9.2 for $\alpha_{\rm S}$, using $\alpha_{\rm EM}(m_c) = 1/133.3$ [96], we obtain

$$\alpha_{\rm S}(m_c) = 0.397^{+0.030}_{-0.040}, \tag{9.3}$$

where the errors have been obtained by varying the value of $\Gamma(\eta_c)$ within its experimental errors. As seen in Eq. 9.2, the first order radiative correction is nearly a factor of two. This casts serious aspersion on this result.

2) It is also possible to make a comparison between the $\gamma\gamma$ and the gg partial widths of η_c. In doing so, we do not have to make the assumption required above (that the η_c and J/ψ wave functions at the origin are exactly equal). From Sec. 2.5, the prediction for the ratio of $\gamma\gamma$ to gg partial widths for η_c is:

$$\frac{\Gamma(\eta_c \to \gamma\gamma)}{\Gamma(\eta_c \to gg)} = B(\eta_c \to \gamma\gamma) = 2.1 \pm 0.4^{+1.1}_{-0.5} \times 10^{-4} = \frac{8{\alpha_{\rm EM}}^2}{9{\alpha_{\rm S}}^2} \times \frac{(1-3.4\alpha_{\rm S}/\pi)}{(1+4.8\alpha_{\rm S}/\pi)}. \tag{9.4}$$

Solving the above equation, we find

$$\alpha_{\rm S}(m_c) = 0.32^{+0.04}_{-0.05}. \tag{9.5}$$

9.2 α_S as determined by χ_2 data

We can also derive a value of $\alpha_S(m_c)$ using our results for χ_2 Like η_c, χ_2 decays via annihilation into two gluons and into two photons. However, χ_2 also has a substantial branching fraction for radiative decay, $B(\chi_2 \to \gamma J/\psi) = 13.5 \pm 1.1\%$. [2] The gg width is thus

$$\begin{aligned} \Gamma(\chi_2 \to gg) &= \Gamma(\chi_2) - \Gamma(\chi_2 \to \gamma J/\psi) - \Gamma(\chi_2 \to \gamma\gamma) \\ &\simeq 0.865 \pm 0.011 \times \Gamma(\chi_2), \end{aligned} \tag{9.6}$$

using the known values of $\Gamma(\chi_2)$, $B(\chi_2 \to \gamma J/\psi)$ and our measured value of $\Gamma(\chi_2 \to \gamma\gamma)$. Using the above relation for $\Gamma(\chi_2 \to gg)$ in the ratio prediction from pQCD,

$$\frac{\Gamma(\chi_2 \to \gamma\gamma)}{\Gamma(\chi_2 \to gg)} = \frac{0.343^{+0.053+0.043}_{-0.051-0.038}\ \mathrm{keV}}{1.71 \pm 0.16\ \mathrm{MeV}} = \frac{8{\alpha_{EM}}^2}{9{\alpha_S}^2} \times \frac{(1 - 5.3\alpha_S/\pi)}{(1 - 2.2\alpha_S/\pi)} \tag{9.7}$$

Solving this equation, we obtain:

$$\alpha_S(m_c) = 0.361^{+0.023}_{-0.020}. \tag{9.8}$$

As noted in the previous chapter, Bodwin, Braaten and Lepage [96] (hereafter, BBL) have made a different calculation for the relationship between P state hadronic widths. Their procedure leads to somewhat different predictions for P-state charmonium decay rate ratios.

In the BBL formalism, the following relationship exists between the hadronic widths (denoted by $\Gamma(\chi_J \to LH)$ of χ_1 and χ_2, and the two-photon width of χ_2. [96]

$$\begin{aligned} \frac{\Gamma(\chi_2 \to \gamma\gamma)}{\Gamma(\chi_2 \to LH) - \Gamma(\chi_1 \to LH)} &= \frac{\Gamma(\chi_2 \to \gamma\gamma)}{(0.865 \pm 0.011) \times \Gamma(\chi_2) - (0.727 \pm 0.016) \times \Gamma(\chi_1)} \\ &= \frac{8{\alpha_{EM}}^2}{9{\alpha_S}^2} \end{aligned} \tag{9.9}$$

Using the measured values of $\Gamma(\chi_2)$ as above and $\Gamma(\chi_1) = 0.88 \pm 0.14$ MeV from E760 [101], we obtain

$$\alpha_{\rm S}(m_c) = 0.394^{+0.043}_{-0.035}. \tag{9.10}$$

Huang and Chao [97] extend the BBL result above to the next order in $\alpha_{\rm S}$, obtaining

$$\frac{\Gamma(\chi_2 \to \gamma\gamma)}{\Gamma(\chi_2 \to LH) - \Gamma(\chi_1 \to LH)} = \frac{8{\alpha_{\rm EM}}^2}{9{\alpha_{\rm S}}^2} \times \frac{(1 - 5.33\alpha_{\rm S}/\pi)}{(1 - 5.17\alpha_{\rm S}/\pi)}. \tag{9.11}$$

U This leads to

$$\alpha_{\rm S}(m_c) = 0.383^{+0.038}_{-0.032}. \tag{9.12}$$

9.3 Comparison to Other Results for $\alpha_{\rm S}$

As mentioned earlier, there are very few measurements of $\alpha_{\rm S}(q)$ in the low energy region near $q = m_c$. One of them is certainly worth mentioning, because of its proximity. It comes from measurements of the hadronic decays of τ, for which $q = m_\tau = 1.78$ GeV. By comparing the semileptonic decay of τ, $(\tau \to \nu_\tau + \ hadrons)$ to the leptonic decay $(\tau \to \nu_\tau + l + \bar{\nu}_l)$ decay rates, a value of $\alpha_{\rm S}$ may be obtained. The PDG [2] summarizes the results of these measurements as $\alpha_{\rm S}(m_\tau^2) = 0.35 \pm 0.03$, which is consistent with our result.

It is also possible to compare our measurement of $\alpha_{\rm S}(m_c)$ to the world average of measurements of $\alpha_{\rm S}$ by extrapolating our measurement to the mass of the Z boson. We can use Eq. 9.1 to scale our result from $q_0 = m_c$ to an equivalent value at the standard $q = M(Z^0)$.

The average of our η_c and χ_2 results calculated using the prescription of Ref. [23] is 0.354 ± 0.018. If we insert $n_f = 3$ and our result into Eq. 9.1, we find that our measurement of $\alpha_s(m_c)$ corresponds to $\alpha_s(M(Z^0)) = 0.119^{+0.005}_{-0.008} \pm 0.007$, assuming that $m_c = 1.5$ GeV. The first error corresponds to our error of 0.018 on $\alpha_s(m_c)$, while the second corresponds to the errors on the PDG averages $\Lambda^{(4)}_{\overline{MS}} = 305 \pm 25 \pm 50$ MeV, and $\Lambda^{(5)}_{\overline{MS}} = 219^{+25}_{-23}$ MeV. Our result is completely consistent with the world average as stated by PDG98 of $\alpha_s(M_Z) = 0.119 \pm 0.002$. [2] Our measurement of $\alpha_s(m_c)$,

$$\alpha_s(m_c) = 0.354 \pm 0.018, \tag{9.13}$$

is shown as an open circle in Fig. 9.1 in which we have assumed $m_c = 1.5$ GeV. The other experimental measurements of α_s in the figure, and Table 9.1 in which we have listed our result, are taken from a compilation by Bethke. [102].

Table 9.1: Measurements of $\alpha_s(Q)$. The first is our result, while the others are taken from the compilation in Ref. [102].

Process	Q (GeV)	$\alpha_s(Q)$
$\bar{p}p \to c\bar{c} \to \gamma\gamma$	1.5	0.354 ± 0.018
DIS	1.58	$.375\ ^{+.062}_{-.081}$
DIS	1.73	$.295\ ^{+.092}_{-.073}$
τ decay	1.78	$.339^{+.021}_{-0.21}$
DIS	4.1	$.223\ ^{+.009}_{-.009}$
DIS	4.13	$.220\ ^{+.027}_{-.027}$
DIS	5.0	$.215\ ^{+.016}_{-.016}$
DIS	7.1	$.180\ ^{+.014}_{-.014}$
$e^+e^- \to$ *hadrons*	10.52	$.20\ ^{+.06}_{-.06}$
$\bar{p}p \to b\bar{b} + X$	20.0	$.145\ ^{+.018}_{-.019}$
e^+e^- event shapes	22.0	$.161\ ^{+.016}_{-.011}$
$\bar{p}p \to \gamma X$	24.2	$.137\ ^{+.017}_{-.014}$
$e^+e^- \to$ *hadrons*	34.0	$.146\ ^{+.031}_{-.026}$
e^+e^- event shapes	35.0	$.145\ ^{+.012}_{-.007}$
e^+e^- event shapes	44.0	$.139\ ^{+.010}_{-.007}$
e^+e^- event shapes	58.0	$.132\ ^{+.008}_{-.008}$
$e^+e^- \to Z^0$	91.2	$.122\ ^{+.005}_{-.005}$
e^+e^- event shapes	91.2	$.122\ ^{+.006}_{-.006}$

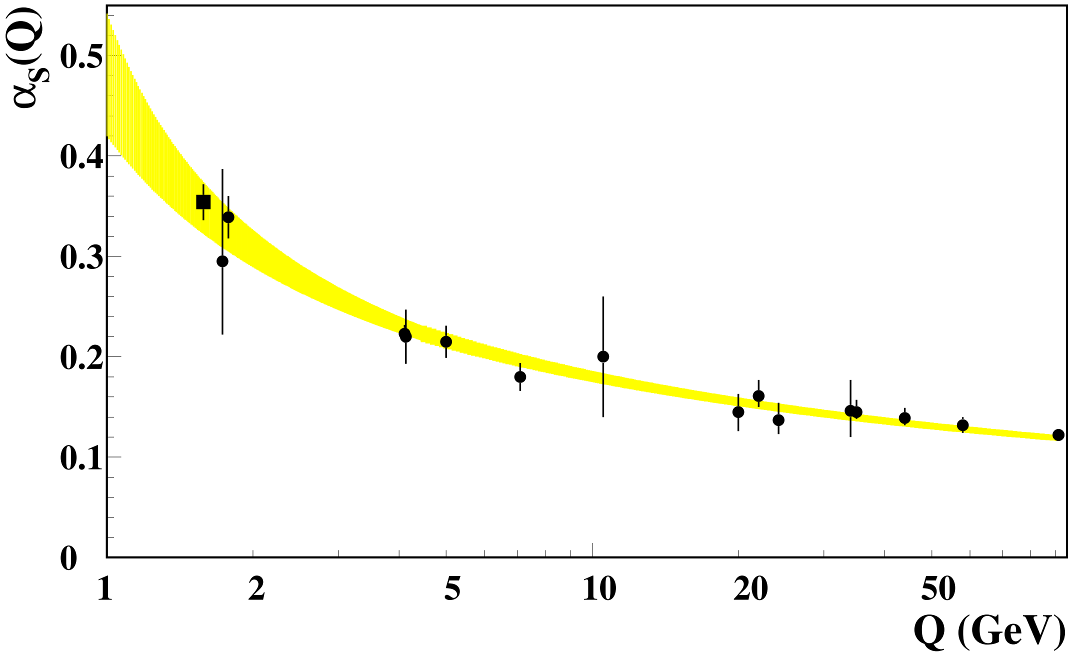

Figure 9.1: Measurements of $\alpha_s(Q)$ from various experiments. [102] The E835 result is indicated by the open circle. The shaded band indicates the evolution of the PDG average value of $\alpha_s(m_Z) = 0.119 \pm 0.002$ from $Q = 91.2$ GeV down to $Q = 1$ GeV, using $\Lambda^{(4)}_{\overline{MS}} = 305 \pm 25 \pm 50$ MeV, and $\Lambda^{(5)}_{\overline{MS}} = 219^{+25}_{-23}$ MeV.[2] The thickness of the band is determined by the errors on $\Lambda^{n_f}_{\overline{MS}}$.